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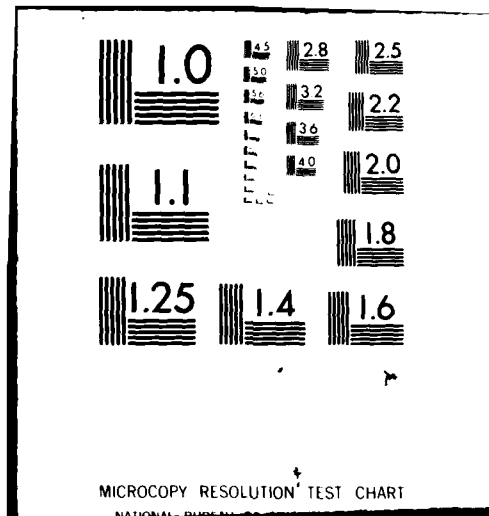
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**ANALYSIS OF COMPOSITE LAMINATES AND**  
**FIBRE COMPOSITE REPAIR SCHEMES**

by  
**R. JONES**

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STRUCTURES NOTE 465 ✓

⑥ **ANALYSIS OF COMPOSITE LAMINATES AND  
FIBRE COMPOSITE REPAIR SCHEMES**

by

⑩ R. JONES

SUMMARY

*In recent years several advanced finite element methods have been developed for the analysis of laminated composites; these take into account the membrane, bending, and interlaminar stresses. Similarly, finite element methods have also been developed for the analysis of structures repaired with a bonded overlay of fibre composite material. The present paper discusses these methods and indicates how the finite element method developed for the analysis of structural repairs is connected to those methods specifically developed for the analysis of composite laminates.*

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## 16. ABSTRACT

*In recent years several advanced finite element methods have been developed for the analysis of laminated composites; these take into account the membrane, bending, and interlaminar stresses. Similarly, finite element methods have also been developed for the analysis of structures repaired with a bonded overlay of fibre composite material. The present paper discusses these methods and indicates how the finite element method developed for the analysis of structural repairs is connected to those methods specifically developed for the analysis of composite laminates.*

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## NOTATION

$G_a$	Shear modulus of the adhesive.
$G$	Shear modulus of a composite ply.
$G_0, G_s$	Transverse shear moduli of the overlay and sheet respectively.
$\tau_{zx}, \tau_{zy}$	Transverse shear stresses.
$t_s, t_a, t_0$	Thicknesses of the sheet, adhesive and overlay respectively.
$u^l, v^l$	In plane displacements in the $l$ th ply.
$t$	Distance between mid-surface of adjacent plies.
$u_0, v_0; u_s, v_s$	In plane displacements in the overlay and the sheet respectively.

## 1. INTRODUCTION

In recent years a number of different approaches [1-14] have been developed for the analysis of laminated composites. We begin by describing these methods in some detail paying particular attention to the need to accurately determine the interlaminar stresses developed in a laminate. Following this discussion we examine the methods developed for the analysis of adhesively bonded metallic, or fibre composite, repairs to damaged structures [27-52].

We then show that the mathematical models developed in [2, 3] for the analysis of laminated composites subjected to in-plane loading only, and the models developed in [27, 28, 29] for the analysis of bonded repair schemes are very similar and may be made identical.

## 2. INTERLAMINAR STRESSES

In classical laminate theory no account is taken of the interlaminar shear stresses  $\tau_{zx}$  and  $\tau_{zy}$  or the peel stresses  $\sigma_z$ . Rather, only the stresses in the plane of the laminate, i.e.  $\sigma_x$ ,  $\sigma_y$ , and  $\tau_{xy}$ , are considered. Accordingly classical laminate theory is incapable of providing predictions for some of the stresses that may cause failure of a composite laminate. One of the earliest attempts to overcome this shortcoming, inherent in classical laminate theory, was the work of Pipes and Pagano [1] who undertook a detailed three-dimensional stress analysis. Following this work Levy and co-workers [2, 3] developed a finite element method, based to some extent on the earlier work of Puppo and Evensen [4], which took the interlaminar shear stresses developed in the composite into account. This method, which is applicable for in-plane loading only, separates the membrane and interlaminar properties of a laminated composite by using alternating orthotropic membrane elements and isotropic shear elements as shown in Figure 1. The orthotropic segments carry in-plane stresses only while the shear segments only carry the interlaminar (i.e. transverse) shear stresses; for full detail's see [2, 3].

In the shear segments the transverse shear stresses  $\tau_{zx}$  and  $\tau_{zy}$  are related to the displacements  $u^l$ ,  $v^l$ , and  $u^{l-1}$ ,  $v^{l-1}$  in the  $l$ th and  $l-1$ th layers respectively by the formulae

$$\tau_{zx} = (u^{l-1} - u^l)/G t \quad (1)$$

$$\tau_{zy} = (v^{l-1} - v^l)/G t \quad (2)$$

where  $t$  is the distance between the midsurface of the  $l$ th and  $l-1$ th layers. Here we have used the notation given in [3] with  $G$  being the transverse shear modulus of the composite. Indeed in [2, 3] it was assumed that  $G_{zx} = G_{zy} = G$ . This approach is very simple to use and, as shown in [3], is easily extended so as to allow for inelastic effects. Furthermore in the case of a ( $\pm 45$ )<sub>s</sub> laminate subjected to uniaxial strain it was shown to give identical results to those given in [1] which were obtained from a full three-dimensional analysis.

Whilst the above methods were being developed research was also underway into developing sophisticated plate bending elements [6, 7] which allow for the transverse shear deformation occurring in laminated composites. These elements were shown to give excellent agreement with known exact solutions obtained from reference [8]. Indeed these approaches have been widely accepted. They have been built into a variety of commercially available packages and implemented on the main frame computers at a number of aerospace companies, e.g. [9].

Although these approaches have been very successful they nevertheless retain the disadvantage, inherent in the analysis presented in [2, 3, 6, 7], that they neglect the "peel" stress  $\sigma_z$ . This stress may in certain circumstances be a prime design variable. Hence several fully three-dimensional composite elements have been developed. Some of these elements are described in references [5, 10]. Most work very well and yield results in agreement with known analytical solutions. One disadvantage of these three-dimensional elements is the size and the band width of the stiffness matrix (and hence the cost of an analysis) which results when attempting to model



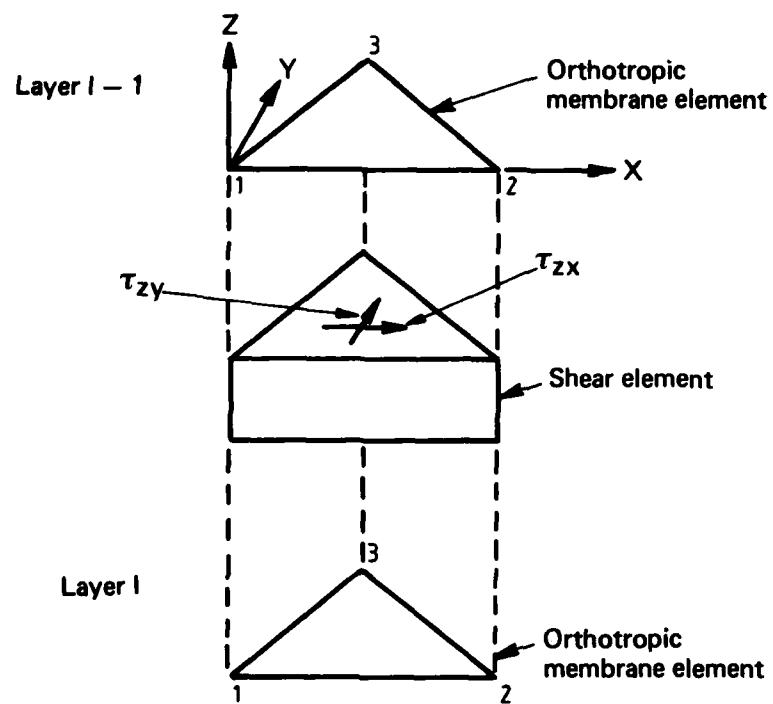


FIG. 1 COMPOSITE ELEMENT

a realistic fibre composite structure. Hence there is a tendency to use elements similar to those developed in [6, 7] whenever possible in preference to the fully three-dimensional elements. To some extent this trend has been encouraged by references [11, 12] which indicate that, even when the peel stresses are important, they may in certain circumstances be accurately estimated from a two-dimensional stress analysis. Furthermore, although it has been shown that significant interlaminar stresses exist around holes [3, 13, 14, 15], it has also been established [16, 17, 18, 19] that a workable failure criterion exists for the failure of composites containing holes or cracks and that this criterion only needs a knowledge of the two-dimensional stress state around the crack or hole. However this is not thought to be true for the growth of disbands between the layers of a laminated composite. Indeed references [20, 21], which specifically deal with the definition and modelling of flaws in composite laminates, when dealing with disbands pay particular attention to the interlaminar stresses. As a result the behaviour of disbands in composite laminates requires, unlike cracks or holes, a detailed knowledge of the interlaminar stresses. This in turn requires either the fully three-dimensional elements [5, 10], the plate bending elements [6, 7]; or the "shear" elements [2, 3] to be used. However the simple "shear" elements should only be used when the stresses in the laminate are primarily membrane stresses. This is often the case in the wing skins of aerospace vehicles.

Whilst, in the above discussion, a number of elements have been described, these are by no means all of the available elements. Other approaches worthy of note are given in references [22, 23, 24, 25, 26]. Of these the work of Reddy [24] is particularly valuable in as much as it compares the effects that reduced integration, mesh size and changing interpolation functions within an element have on the stresses for the case of plate bending elements with shear deformation.

### 3. ADHESIVE ELEMENT

A similar approach to that of Levy *et al.* [2, 3] was used in [27, 28, 29] for the development of an "adhesive" element for the analysis of adhesively bonded repairs to thin metal or composite sheets and where the overlay (repair) is a composite laminate.

In this approach it is assumed that the in-plane displacements vary quadratically through the thickness of the sheet and the overlay and linearly through the thickness of the adhesive. The resultant distribution of the transverse shear stresses through the structure is identical with that used in [30] for the analysis of sandwich structures with thin faces and is in close agreement with the stress distribution obtained in [1] from a full three-dimensional stress analysis. With these assumptions [27, 28, 29] develop the following expressions for the adhesive shear stresses  $\tau_{zx}$  and  $\tau_{zy}$ , for the case of a repair on one side of the sheet only, viz.,

$$\tau_{zx} = (u_0 - u_s)/(t_a/G_a + 3t_0/8G_0 + 3t_s/8G_s) \quad (3)$$

$$\tau_{zy} = (v_0 - v_s)/(t_a/G_a + 3t_0/8G_0 + 3t_s/8G_s) \quad (4)$$

Here  $t_a$ ,  $t_0$  and  $t_s$  are the thicknesses of the adhesive, overlay and sheet respectively while, to enable a comparison with equations (1) and (2), we have considered the case when  $G_{zx} = G_{zy} = G_0$  in the overlay,  $G_{zx} = G_{zy} = G_s$  in the sheet and have defined  $G_a$  as the shear modulus of the adhesive. In addition we have denoted the  $(x, y)$  displacements in the sheet as  $(u_s, v_s)$  and the corresponding displacements in the repair as  $(u_0, v_0)$ .

This approach has been shown in [29] to give strains, on the surface of a repair to a circular cut out and a crack in a metallic sheet, in excellent agreement with measured strains. It has also been used [31, 32] in the design of repairs to the lower wing skin of Mirage III-o aircraft in service with the Royal Australian Air Force. These repairs have been installed on the Mirage fleet and to date the stresses predicted in [31, 32] have been validated by the inservice performance of the repair and by a series of laboratory tests [32] carried out at the Aeronautical Research Laboratories, Australia. For a detailed account of other bonded repair schemes developed at the Aeronautical Research Laboratories the reader is referred to [33].

In the above we have been primarily concerned with a particular method of analysis which allows for shear deformation in the adhesive, repair and sheet. However other methods have also been developed [34-40] which do not allow for shear deformation in the repair or the sheet. As a result, as stated in [34, 38] these methods underestimate the stress intensity factors

of cracks in the repaired structure and overestimate the adhesive shear stress (which is basically an interlaminar stress). Nevertheless they are very useful contributions to repair technology.

The works of Hart-Smith [37] and Swift [36] are particularly interesting since, unlike earlier work, they allow the adhesive to behave plastically. Let us now turn our attention to the design of bonded repair schemes. Detailed design charts have been presented by Davis [41] for the repair of cracks using bonded stringers. Design charts have also been given by Jones *et al.* [28] for the repair of cracked aluminium sheets with a unidirectional boron-epoxy laminate for a crack 38.1 mm long and where the repair completely covers the crack. However the scope of this work has recently been extended by Rose [42] who showed that for the range of thicknesses considered in [28] the value of the stress intensity factor, after repair, remains constant for all crack lengths greater than a critical length which is approximately 5 mm. This means that the design charts given in [28] for a 38.1 mm crack may now be used for all crack lengths greater than 5 mm.

Another design study into the bonded repair of cracks is given by Ratwani in [43]. This study and that of [28] are complementary in as much as whereas in [28] the authors considered the stress intensity factor, the maximum adhesive stress and the peak fibre stresses as the key design quantities reference [43] concentrates on the observed crack growth rate after repair which is an area omitted in [28]. Another important feature of Ratwani's work is that in [34] he gives a simple formulae for the neutral axis offset effect which occurs when repairing a structure on one side only.

So far we have only been considering the use of fibre composite material to repair metallic structures. However the repair of damage to composite structures is an important consideration [44] and is now receiving considerable attention. A detailed discussion of the actual repair procedures for composites is given in [45], which is specifically concerned with sandwich structures. Several theoretical investigations into the repair of composites have also been undertaken [46, 47, 48, 49]. References [46, 47, 49] are primarily concerned with the repair of holes in composites while [48], using an extension of the bonded element developed in [27], considers the repair of cracks and holes. Whereas [46] only considered a bonded metallic repair, [47, 48] consider both composite and metallic repairs and conclude that for holes in composites a metallic repair has certain advantages over a composite repair scheme. This is not true for the repair of cracks in composites as shown in [48]. Reference [49] is valuable both for the comparison between theory and experiment which it gives and also for the list of references which it contains.

Limited research [50, 51, 52, 33] has been undertaken into the repair of surface cracks or cracks in thick metal sections. The only known such repair, actually in use, is the repair to cracks in the main landing wheel of the AerMacchi jet trainer in service with the Royal Australian Air Force. This repair was developed at the Aeronautical Research Laboratories, Australia and is fully documented in [33]. The development of a proposed similar repair for use on main landing wheels of Mirage III aircraft, in service with the Royal Australian Air Force, is described in [50]. In [50] it is shown that the repair of surface cracks in thick sections by means of a bonded overlay of composite material is really only applicable if the cracking is primarily caused by inclusions and/or by high residual stresses rather than by the externally applied load. Reference [51] is concerned only with the repair of surface scratches to thin sheets and makes use of the reduction in the stress intensity factor obtained for a through crack using the two-dimensional analysis methods to predict the growth rates of a repaired surface scratch. These predictions are in reasonable agreement with experimentally determined crack growth rates. The remaining work, [52], is concerned with the variation, along the crack front, of the stress intensity factor for a sandwich structure with a through crack in one face only.

#### 4. A UNIFIED APPROACH

In the previous two Sections we saw how a large amount of work has been undertaken into the analysis of fibre composites and fibre composite repair schemes. In the beginning of Section 2 we paid particular attention to the simple "shear" elements developed by Levy and coworkers [2, 3] for the analysis of the interlaminar stresses developed in composite laminates. We subsequently saw in Section 3 that an adhesive element very similar to the "shear" element of Levy has been developed [27, 28, 29] to account for the transverse shear stresses developed

in the adhesive layer for bonded repairs to metallic or composite structures. Yet despite the similarity of these two approaches it appears, at first glance, that the expressions for the shear stresses used in [2, 3] and [27, 28, 29] differ due to the difference in the denominators in equations (1), (2) and (3), (4). However it will now be shown that equations (3), (4) may also be used to model the shear stress distribution in a composite laminate.

To do this we first interpret  $t_a$  as the "interlaminar resin layer" which separates two plies, or bundles of plies of composite material, the thickness of these plies, or bundles of plies, being  $t_s$  and  $t_0$  respectively. With these definitions the interlaminar shear stresses in the "interlaminar resin layer" may, using the approach outlined in [27, 28, 29], now be obtained from equations (3) and (4).

Let us now compare this model with that of Levy *et al.*, as given by equations (1) and (2) for the case of the  $(\pm 45)_s$  laminate subjected to uniaxial extension which was initially considered in [2]. In this case we will compare the interlaminar shear stresses in the resin separating the  $+45^\circ$  and the  $-45^\circ$  plies. If we consider, as in [2], that these plies have the same thickness (i.e.,  $t_s = t_0$ ) and, in accordance with [1, 2], the same transverse shear moduli (i.e.,  $G_0 = G_s = G$ ) then the separation of the mid-surface of the plies is equal to  $t_a + t_0$  and equations (1) and (2) reduce to

$$\tau_{zx} = G(u_0 - u_s)/(t_a + t_0) \quad (5)$$

$$\tau_{zy} = G(v_0 - v_s)/(t_a + t_0) \quad (6)$$

Here in order to enable a simple comparison between the two approaches we have adopted the notation  $u^l = u_0$ ,  $u^{l-1} = u_s$  etc.

In general it is difficult to exactly determine  $t_a$ . However, it has recently been suggested [52], and confirmed by examination under the microscope that  $t_a$ , the thickness of the interlaminar resin layer, is approximately one tenth the thickness of an individual ply. Thus in our case  $t_a = 0.1 t_0$  and so equations (5), (6) reduce to

$$\tau_{zx} = G(u_0 - u_s)/1.1 t_0 \quad (7)$$

$$\tau_{zy} = G(v_0 - v_s)/1.1 t_0 \quad (8)$$

Alternatively, if we use equations (3) and (4) to determine the shear stresses  $\tau_{zx}$  and  $\tau_{zy}$  we obtain

$$\tau_{zx} = G(u_0 - u_s)/t_0(0.75 + t_a G/G_a t_0) \quad (9)$$

$$\tau_{zy} = G(v_0 - v_s)/t_0(0.75 + t_a G/G_a t_0) \quad (10)$$

We thus see that one approach yields  $1.1 t_0$  as the denominator while the other yields  $t_0(0.75 + t_a G/G_a t_0)$ .

The value of  $G_a$  given in [53] for a typical epoxy resin is 1.28 GPa while in the problem under consideration the value of  $G$  used in [2, 3] is 4.98 GPa. These values give

$$0.75 + t_a G/t_0 G_a = 1.13$$

As a result we see that in practice there is very little difference between the two approaches. Indeed since the value of  $t_a$  is to some extent uncertain it would be best to set the ratio  $t_a G/t_0 G_a$  to be 0.35, where  $t_0$  is the thickness of an individual ply, so that the two models would coincide exactly.

## 5. CONCLUSION

In this paper we have outlined the various finite element methods which are available for the analysis of composite laminates and fibre composite repair schemes. In addition we have attempted to indicate when a detailed analysis is required and when simple methods will suffice. We have also presented a summary of the developments in use of bonded repair schemes and have provided an insight into the manner in which the present numerical methods, used to design bonded repairs, are linked to the numerical methods used to analyse composite laminates.

## REFERENCES

1. R. B. Pipes and N. J. Pagano—Interlaminar stresses in composite laminates under uniform axial extension, *J. Composite Materials*, **4**, 538-548 (1970).
2. G. Isakson and A. Levy—Finite element analysis of interlaminar shear in fibrous composites, *J. Composite Materials*, **5**, 273-275 (1971).
3. A. Levy, H. Armen Jr and J. Whiteside—Elastic and plastic interlaminar shear deformation in laminated composites under generalized plane stress, *Proceedings 3rd Conference on Matrix Methods of Structural Analysis*, Wright Patterson AFB, 1971.
4. A. H. Puppo and H. A. Evensen—Interlaminar shear in a laminated composite under generalized plane stress, *J. Composite Materials*, **40**, 204-220 (1970).
5. R. M. Barker, Fu-Tien Lin and J. R. Dana—Three-dimensional finite element analysis of laminated composites, *Computers and Structures*, **2**, 1013-1029 (1972).
6. C. W. Pryor Jr and R. M. Barker—A finite element analysis including transverse shear effects for applications to laminated plates, *J. AIAA*, **9**, 5, 912-917 (1971).
7. S. T. Mau, P. Tong and T. H. H. Pian—Finite element solutions for laminated thick plates, *J. Composite Materials*, **6**, 304-311 (1972).
8. N. J. Pagano—Exact solutions for composite laminates in cylindrical bending, *J. Composite Materials*, **3**, 398-411 (1969).
9. D. C. Connel—Development of a computer program for the finite element analysis of laminated plates including transverse shear effects, *Westlands Helicopters Limited, Research Paper No. 480*, August 1975.
10. O. C. Zienkiewicz—The finite element method in Engineering Science, McGraw Hill, London 1971.
11. N. J. Pagano and R. B. Pipes—Some observations on the interlaminar strength of composite laminates, *Int. J. Mech. Sci.*, **15**, 679-688 (1973).
12. D. O. Stalnaker and W. W. Stinchcomb—Load history—edge damage studies in two quasi-isotropic graphite epoxy laminates, *ASTM STP674*, 620-641 (1979).
13. E. F. Rybicki and D. W. Schmueser—Three-dimensional finite element analysis of laminated plates containing a circular hole, *AFML-TR-76-92*, 1976.
14. S. Tang—Interlaminar Stresses around circular cutouts in composite plates under tension, *J. AIAA*, **15**, 1631-1637 (1977).
15. R. M. Barker, J. R. Dana and C. W. Pryor Jr.—Stress concentrations near holes in laminates, *J. Engn and Mechanics*, **100**, 477-488 (1974).
16. R. J. Nuismer and J. M. Whitney—Uniaxial failure of composite laminates containing stress concentrations, *ASTM-STP593*, 117-141 (1975).
17. R. J. Nuismer and J. D. Labor—Application of the average stress failure criterion: Part II—Compression, *J. Composite Materials*, **13**, 49-60 (1979).
18. G. Caprino, J. C. Halpin and L. Nicolais—Fracture mechanics in composite materials, *Composites*, 223-227 (1979).

19. W. P. Witt III, A. N. Palazotto and H. T. Hahn—Numerical and experimental comparison of notch tip stresses in a laminated graphite/epoxy plate. *Proceedings AIAA/ASME 19th Structures, Structural dynamics and materials conference*, Bethesda, 1978, 262–269 (1978).
20. R. L. Ramkumar, S. V. Kulbarni and R. B. Pipes—Definition and modelling of critical flaws in graphite fiber reinforced epoxy resin matrix composite materials, *Materials Sciences Corporation*, 1978, report number NADC-76228-30 contract number N6229-77-G0092.
21. D. J. Wilkins—Damage tolerance modeling for composite structures, *General Dynamics, Fort Worth Division*, Report C.SE-108, January 1980.
22. A. S. Mawenya and J. D. Davis—Finite element bending analysis of multilayer plates, *Int. J. Numer. Meth. Engng*, **8**, 215–225 (1974).
23. J. N. Reddy—A penalty plate bending element for the analysis of laminated anisotropic composite plates, *Int. J. Numer. Meth. Engng* (to appear).
24. J. N. Reddy—A comparison of closed form and finite element solutions of thick, laminated anisotropic rectangular plates, *Dept of the Navy, Office of Naval Research*, Report OV-AMNE-79-19, December 1979.
25. S. Ahmad, B. M. Irons and O. C. Zienkiewicz—Analysis of thick and thin shell structures by curved finite elements, *Int. J. Num. Meth. Engng*, **2**, 419–451 (1970).
26. S. C. Panda and R. Natarajan—Finite element analysis of laminated composite plates, *Int. J. Num. Meth. Engng*, **14**, 69–79 (1979).
27. R. Jones and R. J. Callinan—Finite element analysis of patched cracks, *J. Struc. Mechanics*, **7**, 2, 107–130 (1979).
28. R. Jones and R. J. Callinan—A design study in crack patching, *J. Fibre Science and Technology* 1980 (in press) also see *Aeronautical Research Laboratories, Structures Report* 376, July 1979.
29. R. A. Mitchell, R. M. Wodey and D. J. Chivirut—Analysis of composite reinforced cutouts and cracks, *J. AIAA*, **13**, 744–749 (1975).
30. H. G. Allen—Analysis and design of structural sandwich panels, *Pergamon Press*, Oxford, 1969.
31. A. A. Baker, R. J. Callinan, M. J. Davis, R. Jones and J. G. Williams—Application of b.f.r.p. crack patching to Mirage III aircraft, *Proceedings 3rd Int. Conf. Composite Materials*, Paris, August 1980.
32. R. Jones and R. J. Callinan—Structural design of b.f.r.p. patches for Mirage wing repair, *Aeronautical Research Laboratories, Structures Note* 461, July 1980.
33. A. A. Baker—A summary of work on applications of advanced fibre composites at the *Aeronautical Research Laboratories, Australia*, *Composites* **9**, 11–16 (1978).
34. M. M. Ratwani—Analysis of cracked, adhesively bonded structures, *J. AIAA*, 155–163 (1978).
35. K. Arin—A plate with a crack stiffened by a partially debonded stringer, *Engng Frac. Mech.*, **7**, 173–179 (1975).
36. T. Swift—Fracture analysis of adhesively bonded cracked panels, *J. Engng Materials and Technology*, *Trans A.S.M.E.*, **100**, **1**, 10–15 (1978).
37. L. J. Hart-Smith—Adhesive bond stresses and strains at discontinuities and cracks in bonded structures, *Trans. ASME, J. Engng Materials and Technology*, **100**, **1**, 16–25 (1978).
38. G. Dowrick and D. J. Cartwright—The effect of a circular reinforcing patch on a crack in a uniaxially stressed sheet, *University of Southampton, Engng Materials Laboratory*, Report No. ME/80/2, January 1980.

39. F. Erdogan and K. Arin—A sandwich plate with a part through and a debonding crack, *Engng. Frac. Mech.*, **4**, 449–458 (1972).
40. G. Bartelds—Residual strength characteristics of composite reinforced stiffened panels, National Aerospace Laboratory, Amsterdam, NLR-TR-78025, 1978.
41. M. J. Davis—Repair of aircraft structure using bonded high performance composite materials, M. Eng. Thesis, Victoria Institute of Colleges, Caulfield Institute of Technology, Australia, November 1979.
42. L. R. F. Rose—A cracked plate repaired by bonded reinforcements, submitted to *Int. J. Fracture Mechanics* in 1980.
43. M. M. Ratwani—A parametric study of fatigue crack growth behaviour in adhesively bonded metallic structures, *Trans. ASME, J. Engng Materials and Technology*, **100**, **1**, 46–51 (1978).
44. *Plastics for Aerospace Vehicles, Part 1, Reinforced Plastics*, US Military Handbook 17A.
45. *Structural Sandwich Composites*, US Mil. Handbook 23, 30th December 1968.
46. G. Lubin, S. Dastir, J. Mahon and T. Woodrum—Repair technology for boron-epoxy Structures, Proceedings of the 27th Annual Technical Conference, 1972. Reinforced Plastics/Composites Institute, Section 17-B, pp. 1–12.
47. L. H. Kocher and S. L. Cross—Reinforced cutouts in graphite composite structures, *ASTM STP 497*, 382–395 (1972).
48. R. Jones, R. J. Callinan and K. C. Aggarwal—Stress analysis of adhesively bonded repairs to fibre composite structures, Structures Report, Aeronautical Research Laboratories, Australia (to be published).
49. R. A. Heller and T. Chiba—Alleviation of stress concentration with analog reinforcement, Proceedings 1973 SESA. Spring meeting, Los Angeles, California, 1973.
50. R. Jones and R. J. Callinan—Analysis and repair of flaws in thick structures, Proceedings 5th Int. Conf. on Fracture, Cannes, France, March 1981 (in press).
51. J. D. Labor and M. M. Ratwani—Composite patches for metal structures, Naval Air Development Centre, Pennsylvania, Contract No. N62269-79-C-0271, progress report No. 4, 1st April–31st May 1980.
52. S. N. Alturi and K. Kathiresan—Stress analysis of typical flaws in aerospace structural components using 3-D hybrid displacement finite element method. Proceedings 19th ASME/AIAA Structures, Structural dynamics and Materials conference, Bethesda, 1978, pp. 340–350 (1978).
53. S. S. Wang—An analysis of delamination in angle-ply fibre-reinforced composites, *Trans. ASME, J. Applied Mechanics*, **7**, **1**, 64–71 (1980).

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